# Accumulation of Solid Particles on Single Fibers Exposed to Aerosol Flows

**DEVENDRA T. BAROT, CHI TIEN, and CHIU-SEN WANG**

**Department of Chemical Engineering and Materials Science**
**Syracuse University**
**Syracuse, New York 13210**

The phenomenon of particle accumulation plays an important role in many fluid-particle separation processes, such as gas cleaning by fibrous filters. One approach to understanding the effects of particle accumulation is to study the formation and growth of particle deposits on single collectors as the "building blocks" of a fluid-particle separation device. Using this approach, we can formulate simple theoretical models to predict the effects of important parameters. Further, the theoretical models for single collectors can be tested by comparing the predictions directly with experimental data.

When a single fiber is exposed to a stream of gas carrying solid particles, particle deposits on the fiber tend to grow in a dendritic form. This was first observed by Watson (1946), but no quantitative experimental results were reported until two decades later. In 1966, Billings measured the distribution of dendrite size, that is, the number of particles in each dendrite, as a function of the exposure time during which the fiber was placed normal to an aerosol stream. Among the data for seven single fibers reported by Billings, only two had size distributions determined at more than three different exposure times. And, the first measurement for each of the seven fibers was made after a significant number of dendrites had already formed on it. The data, therefore, do not comprise a sufficiently complete study of the dynamics of dendrite growth during the initial stage of particle accumulation.

The first attempt to model the growth of particle clusters on fibers was made by Radushkevich (1964). His model assumes that a dendrite can be characterized by the number of particles it contains, regardless of the configuration. In 1976, Payatakes and Tien proposed a model which does take into account the effects of dendrite configuration by assuming that the particle chains grow in orderly patterns. Payatakes (1976a, b, 1977) reports further developments of this model.

We have recently developed a general theory for the accumulation of solid particles on a collecting surface that is exposed to a stream of fluid-solid suspension (Tien et al. 1977). The theory is based on the following postulates on the behavior of particles in a dispersed system:

1. In a suspension in which the concentration of particles is uniform macroscopically, the spatial distribution of the particles obeys the Poisson law. Green (1927) has shown this to be valid for aerosols.
2. Given the initial position and velocity, the particle trajectory is determined by the forces acting on the particle.
3. Any macroscopic property of a fluid-particle system (such as the deposition flux of particles on a collecting surface) can be predicted by taking the ensemble average of the corresponding property of a large number of similar systems that have macroscopically identical initial conditions.

Correspondence concerning this note should be addressed to Chiu-sen Wang.

0001-1541-80-2926-0289-$00.75. 

The theory is a general one, and therefore is applicable to any fluid-particle separation system. Unlike the Payatakes-Tien model, the general theory does not assume a priori the location at which the deposit would form, nor the manner in which the deposit would grow. Procedures for simulating the deposition process based on this theory and some results of simulation for spherical collectors have been reported by Wang et al. (1977). Kanaoka et al. (1978) used a similar approach to predict the growth of particle deposits in fibrous filters.

Here, we report data on the dendrite size distribution for deposits of 1.091 and 2.02 $\mu$m particles formed on single fibers approximately 9 $\mu$m in diameter. These data were obtained and analyzed to test the general theory developed by Tien et al. (1977).

## APPARATUS AND METHODS

The experimental set-up (Figure 1) provides a uniform flow of aerosols at velocities of 10 to 50 cm/sec at the test section, where a fiber was placed at right angles to the flow. A Collison nebulizer was used to produce a spray of droplets from liquid suspensions of 1.091 $\mu$m polystyrene spheres or 2.02 $\mu$m polyvinyltoluene spheres. The aerosol thus produced was exposed to a bipolar ion cloud generated by a Kr-85 source, thereby reducing the electrostatic charges on particles to the Boltzmann equilibrium distribution. The neutralized aerosol then entered an aerosol tunnel, where it mixed with dry air to provide the desired air velocity at the test section. The contraction

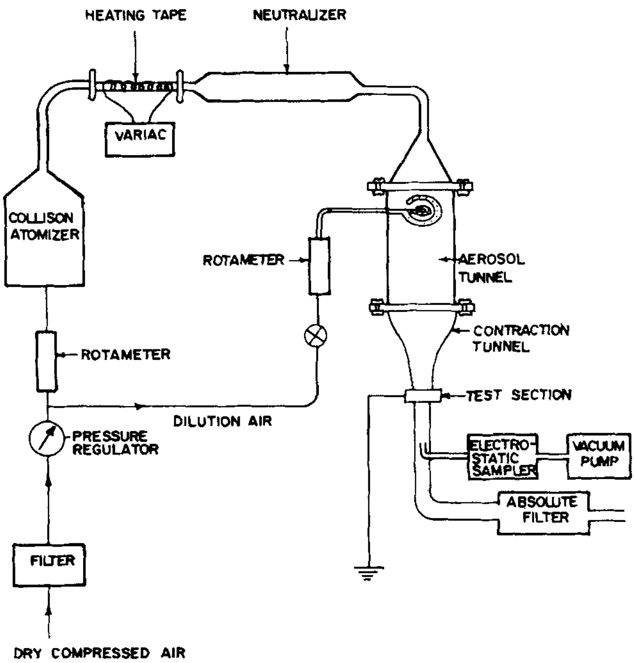


**Figure 1. Experimental apparatus.**

**Table 1. Experimental Conditions**

| Fiber No. | Fiber diameter, μm | Particle diameter, μm | Air velocity, cm/sec | Aerosol concentration part./cm³ | Reynolds number, $Re_f$ | Stokes number, $St$ | Relative size, $N_R$ |
|---|---|---|---|---|---|---|---|
| 6 | 8 | 1.091 | 12.1 | 11,523 | 0.0684 | 0.116 | 0.136 |
| 7 | 8 | 1.091 | 19.9 | 6,809 | 0.106 | 0.191 | 0.136 |
| 8 | 8 | 1.091 | 36.2 | 6,380 | 0.193 | 0.350 | 0.136 |
| 9 | 9 | 2.020 | 12.1 | 558 | 0.0730 | 0.360 | 0.224 |
| 10 | 8.5 | 2.020 | 13.2 | 360 | 0.0746 | 0.410 | 0.238 |
| 11 | 8.5 | 2.020 | 23.4 | 338 | 0.133 | 0.728 | 0.238 |

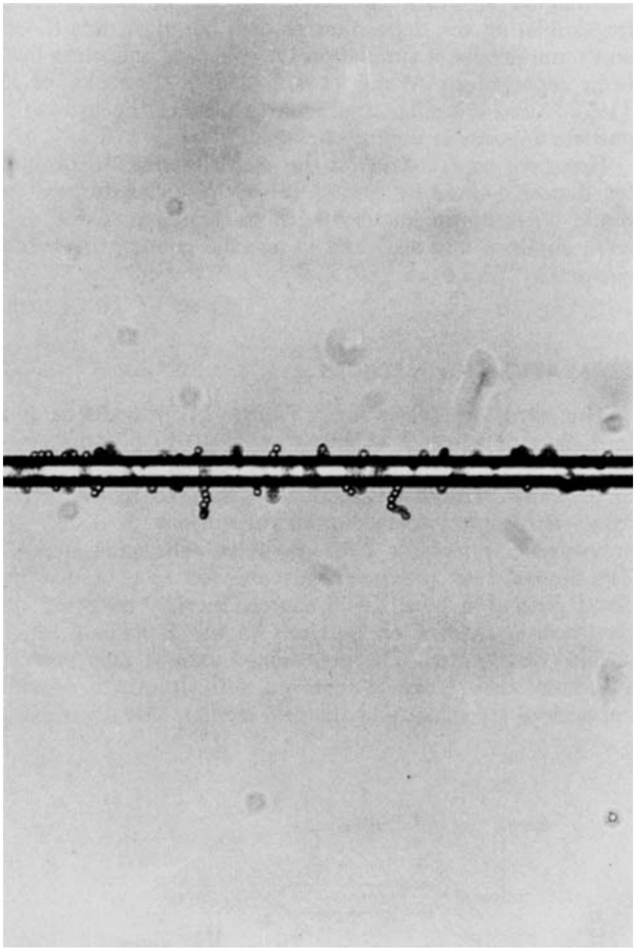

Figure 2. Deposits on a section of fiber No. 11 at $t = 210$ min.

part of the tunnel, fabricated according to the offsets given by Billings (1966), provided a uniform aerosol flow field. A ring of 3.81 cm inside diameter was placed at the end of the aerosol tunnel to hold single glass fibers for experiments. Particle concentration of the aerosol was determined by the number of particles collected by an electrostatic sampler through a probe located 38.1 cm downstream of the test section.

In each run, the fiber was removed with the holder ring from the test section, to observe the particle deposits at time intervals of 15 to 60 min. Number of dendrites and number of particles in each dendrite were counted over two separate fiber sections, each 250 μm long, using an optical microscope. A run was terminated if there were too many particles deposited on the fiber for clear observation. The particle concentration of the test aerosol was determined at the midpoint of each time interval in a run.

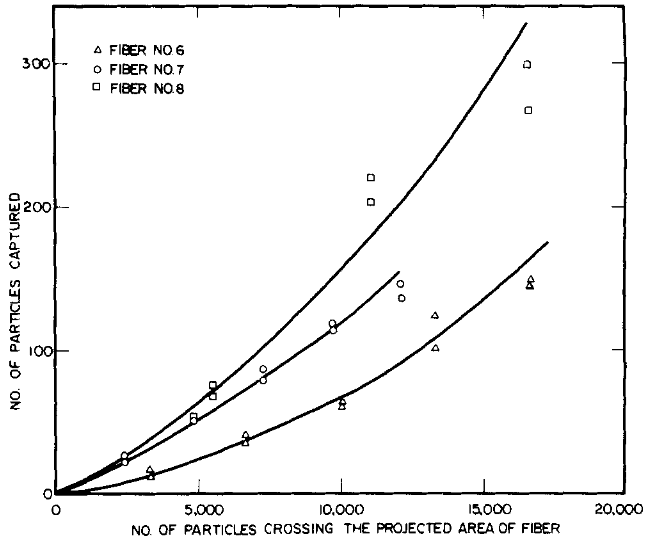


Figure 3. Number of particles captured over 250 μm of a fiber as a function of number of particles crossing the projected area of fiber, for fiber Nos. 6 to 8.

## RESULTS AND DISCUSSIONS

Eleven runs were carried out to study the effects of air velocity and particle size on the structure of deposits and the capture efficiency. The results of the first five runs were rejected because of errors due to leakage of aerosols

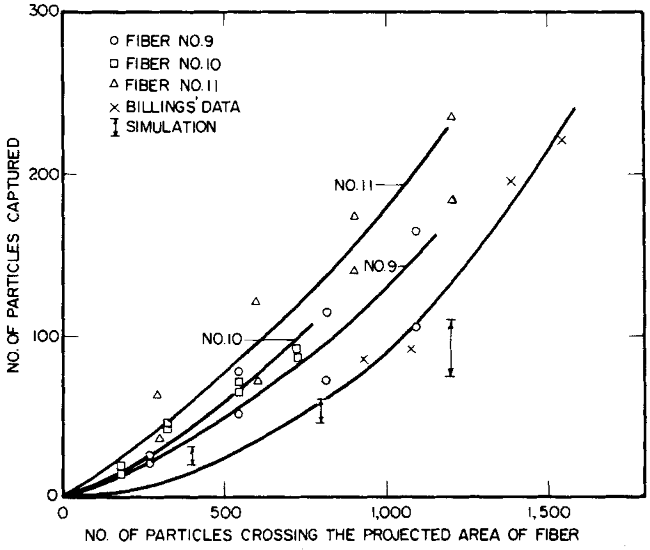


Figure 4. Number of particles captured over 250 μm of a fiber as a function of number of particles crossing the projected area of fiber, for fiber Nos. 9 to 11. Billings' data and results of simulation are included for comparison.

Table 2. Dendrite Size Distributions for Two Sections, Each 250 μm in Length, on Fiber No. 6

| Exposure time, min. | 20 | | 40 | | 60 | | 80 | | 100 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Section | I | II | I | II | I | II | I | II | I | II |
| Dendrite size | | | | | | | | | | |
| 1 | 13 | 15 | 25 | 27 | 30 | 25 | 35 | 34 | 30 | 28 |
| 2 | | 1 | 4 | 5 | 10 | 12 | 18 | 13 | 21 | 19 |
| 3 | | | 1 | 0 | 3 | 3 | 4 | 9 | 6 | 8 |
| 4 | | | | 1 | 1 | 1 | 2 | 3 | 4 | 4 |
| 5 | | | | | | | 1 | 1 | 2 | 3 |
| 6 | | | | | | | 1 | 2 | 3 | 3 |
| 7 | | | | | | | | 0 | 1 | 0 |
| 8 | | | | | | | | 1 | 1 | 1 |
| Total number of dendrites | 13 | 16 | 30 | 33 | 44 | 41 | 61 | 63 | 68 | 66 |
| Total number of particles captured | 13 | 17 | 36 | 41 | 63 | 62 | 102 | 124 | 149 | 147 |
| Number of particles crossing projected area of fiber | 3340 | | 6680 | | 10,020 | | 13,360 | | 16,700 | |

in the apparatus. Table 1 lists the experimental conditions of the later six runs. Data on the dendrite size distributions for two 250 μm sections of Fiber No. 6 are summarized in Table 2. Additional data are reported in Barot's thesis (1977). Figure 2 shows the deposit on a section of Fiber No. 11 after an exposure time of 210 minutes.

In Figures 3 and 4, the number of particles captured over a 250 μm section of fiber is plotted against the number of particles crossing the projected area of the fiber. A smooth curve was drawn by eyeball fitting for the data points of each fiber. Curves are generally concave upwards, indicating that collection efficiency increases with the accumulation of particles. For comparison, Billings' data are included in Figure 4 (Fiber No. 4 in Billings' work, with particle diameter = 1.305 μm, fiber diameter = 9.6 μm, air velocity = 13.8 cm/sec, average particle concentration = 970 part./cm$^3$).

Data in Table 2 can be used to calculate the collection efficiency of a fiber according to the following equation:

$$\eta(t) = \frac{1}{d_f L v c} \frac{dN}{dt}$$

Because the total number of particles crossing the projected area of the fiber in a time period, $t$, is $N_0$ ($= d_f L v c t$), the fiber efficiency can also be written as

$$\eta(t) = \frac{dN}{dN_0}$$

The fiber efficiency is therefore the slope of the curve of $N$ against $N_0$. The collection efficiencies at $t = 0$ calculated from the experimental data are: $3.17 \times 10^{-3}$, $8.5 \times 10^{-3}$, $1.08 \times 10^{-2}$, $6.94 \times 10^{-2}$, $1.57 \times 10^{-1}$ for Fiber Nos. 6, 7, 8, 9 and 11, respectively.

Based on the theory of Tien et al. (1977), Beizaie (1977) carried out a number of numerical simulations for the collection of particles by single fibers. Shown in Figure 4 are the results obtained by Beizaie for $St$ (Stokes number) = 0.614, $N_R$ (relative size) = 0.238, and $Re_f$ (Reynolds number) = 0.112, using the flow field past a cylinder at low Reynolds number derived by Davies (1950). The bars in the figure indicate the range for ten independent simulations. Agreement between the calculations and the experimental data is fair, indicating the theory of Tien et al. is fairly realistic.

## NOTATION

$C$ = Cunningham correction factor
$c$ = particle concentration in an aerosol, part./m$^3$
$d_f$ = fiber diameter, m
$d_p$ = particle diameter, m
$L$ = length of a section of fiber, m
$N$ = number of particles captured over a specified length of fiber in a specified time period
$N_0$ = number of particles crossing the projected area of a specified length of fiber in a specified time period
$N_R$ = relative size; the ratio of the particle diameter to the fiber diameter
$Re_f$ = Reynolds number defined as $d_f v \rho/\mu$
$St$ = Stokes number defined as $C\rho_p v d_p^2/9\mu d_f$
$t$ = exposure time, sec
$v$ = air velocity in the stream, m/sec
$\mu$ = viscosity of air, pascal-second
$\rho$ = density of air, kg/m$^3$
$\rho_p$ = density of particle, kg/m$^3$

## LITERATURE CITED


Barot, D. T., "Formation and Growth of Particle Dendrites on Single Fibers Exposed to Aerosol Flow," M.S. Thesis, Syracuse University, Syracuse, New York (1977).

Beizaie, M., "Deposition of Particles on a Single Collector," Ph.D. Dissertation, Syracuse University, Syracuse, New York (1977).

Billings, C. E., "Effects of Particle Accumulation in Aerosol Filtration," Ph.D. Dissertation, California Institute of Technology, Pasadena, California (1966).

Davies, C. N., "Viscous Flow Transverse to a Circular Cylinder," *Proc. Roy. Soc.* (London), B63, 288 (1950).

Green, H. L., *Phil. Mag.*, 4, 1064 (1927) as quoted in *Particulate Clouds—Dust, Smokes, and Mists*, p. 281, Van Nostrand, New York (1964).

Kanaoka, C., H. Emi, T. Myojyo, "Simulation of Deposition and Growth of Airborne Particles on a Filter," *Kagaku Kogaku Ronbunshu*, (in Japanese), 4, 535 (1978).

Payatakes, A. C., "Model of Aerosol Particle Deposition in Fibrous Media with Dendrite-Like Pattern—Application to Pure Interception During Period of Unhindered Growth," Filtr. Sep., 13, 602 (1976*a*).

Payatakes, A. C., "Model of Dynamic Behavior of a Fibrous Filter—Application to Case of Pure Interception During Period of Unhindered Growth," *Powder Technol.*, 14, 267 (1976*b*).

Payatakes, A. C., "Model of Transient Aerosol Particle Deposition in Fibrous Media with Dendritic Pattern," *AIChE J.*, **23**, 192 (1977).
Payatakes, A. C., and Chi Tien, "Particle Deposition in Fibrous Media with Dendrite-like Pattern: A Preliminary Model," *J. Aerosol Sci.*, **7**, 85 (1976).
Radushkevich, L. V., "Kinetics of Formation and Growth of Aggregates on a Solid Obstacle from a Stream of Colloidal Particles," *Colloid J. U.S.S.R.* (English translation of Kolloid Zh.), **26**, 194 (1964).
Tien, C., C. S. Wang, and D. T. Barot, "Chainlike Formation of Particle Deposits in Fluid-Particle Separation," Science, **196**, 983 (1977).
Wang, C. S., M. Beizaie, and C. Tien, "Deposition of Solid Particles on a Collector: Formulation of a New Theory," *AIChE J.*, **23**, 879 (1977).
Watson, J. H. L., "Filmless Sample Mounting for the Electron Microscope," *J. Appl. Phys.*, **17**, 121 (1946).

*Manuscript received March 7, 1979; revision received May 7, and accepted May 10, 1979.*

# On an Analysis of Draw Resonance by Hyun

**MORTON M. DENN**
**Department of Chemical Engineering**
**University of Delaware**
**Newark, Delaware 19711**

Draw resonance is an oscillatory instability that is sometimes observed in the continuous drawing ("spinning") of polymer filaments. Experimental observations and theoretical understanding of the phenomenon are discussed in recent reviews by Pearson (1976), Petrie and Denn (1976), and Kase and Denn (1978).

An analysis of draw resonance that purports to provide analytical solutions and physical insight has been reported in two papers in this journal by Hyun (1978*a*, *b*). The approach is based on the notion of "throughput waves," and exploits the hyperbolic nature of the governing partial differential equations. Hyun obtains results for the onset of draw resonance that are close to those obtained by other investigators from more classical methods of hydrodynamic stability theory, but there are important unnoted quantitative differences. We show here that Hyun's analysis is incorrect.

## CONTRADICTION

For slow speed, isothermal spinning, the equations of conservation of mass and momentum used by all workers are

$$\text{mass:}\quad \left.\frac{\partial A}{\partial t}\right)_x + \left.\frac{\partial Q}{\partial x}\right)_t = 0 \qquad (1a)$$

$$Q = Av \qquad (1b)$$

$$\text{momentum:}\quad F = \text{function of } t \text{ but not } x \qquad (2)$$

It is simplest to restrict the discussion to a Newtonian fluid, for which we have the further constitutive relation

$$\text{constitutive:}\quad F = 3\eta A \left.\frac{\partial v}{\partial x}\right)_t \qquad (3)$$

This system of equations requires three spatial boundary conditions; the important one for our discussion is imposition of a fixed takeup velocity,

$$v(L, t) = \text{constant} \qquad (4)$$

Hyun has rewritten the continuity equation in a form that emphasizes its hyperbolic nature:

$$\left.\frac{\partial Q}{\partial t}\right)_x + U \left.\frac{\partial Q}{\partial x}\right)_t = 0 \qquad (5)$$

Dr. Denn is presently with the Department of Chemical Engineering, California Institute of Technology, Pasadena, California.

0001-1541-80-2947-0292-$00.75. 

It follows from direct substitution that

$$U = \left.\frac{\partial Q}{\partial t}\right)_x \Bigg/ \left.\frac{\partial A}{\partial t}\right)_x \qquad (6)$$

Hyun has implicitly made the following transformation of independent variables:

$$x, t \rightarrow x, A(x, t) \qquad (7)$$

This transformation is valid only if $\partial A/\partial t)_x$ never vanishes, which is of course impossible in a system exhibiting sustained oscillations. The invalid transformation may be a partial cause of the incorrect solution, but we believe that the root cause is a more serious error discussed later. If we assume however that Equation (7) is valid, then, following Hyun,

$$U = \left.\frac{\partial Q}{\partial A}\right)_x = v + A \left.\frac{\partial v}{\partial A}\right)_x \qquad (8)$$

The second equality in Equation (8) is not used by Hyun, but follows directly from Equation (1b).

The development is now adequate to demonstrate that Hyun's result is incorrect. Through a further series of manipulations on Equation (8) he argues that $U$ is a constant in time and space, given by his Equation (13):

$$\text{Hyun:}\quad U = \frac{v_o(r-1)}{\ln r} \qquad (9)$$

$r$ is the drawdown (area reduction) ratio, which we have generally called $D_R$ and which others have called $E$.

We now suppose that $U$ is indeed a constant, and show that a contradiction results. Equation (8) can be integrated for constant $U$ to give $v$ as a function of the independent variable $A$ at each value of the independent variable $x$:

$$v = \frac{v_o(r-1)}{\ln r} + \frac{K(x)}{A} \qquad (10)$$

$K(x)$ is a constant of integration that may depend on $x$, but not on $A$. At the takeup, $x = L$, the velocity $v(L, t)$ is a constant, Equation (4), equal to $rv_o$. Thus,

$$\frac{K(L)}{A} = \frac{v_o}{\ln r}\{r \ln r - r + 1\} \qquad (11)$$

The right-hand side of Equation (11) is a constant, and $K(L)$ must be independent of $A$. Thus, when $x = L$, $A$ must be a constant; i.e., if Equation (9) is correct, *then the takeup area cannot vary in time and draw resonance*